

AD-A054 314

RENSSELAER POLYTECHNIC INST TROY N Y DEPT OF ELECTRI--ETC F/G 8/2  
APPLICATION OF THE GENERALIZED CHAIN CODING SCHEME TO MAP DATA --ETC(U)  
1978 H FREEMAN AFOSR-76-2937

UNCLASSIFIED

AFOSR-TR-78-0877

NL

| OF |  
AD  
A054314



END  
DATE  
FILMED  
6 -78  
DDC

AFOSR-TR-78-0877

AD A054314

DDC FILE COPY

APPLICATION OF THE GENERALIZED CHAIN CODING SCHEME TO MAP DATA PROCESSING

Herbert Freeman

11/1978

DDC

MAY 26 1978

Rensselaer Polytechnic Institute, Troy, New York 12181

Abstract

The concept of chain coding based on the well known 8-direction coding matrix is generalized to coding schemes involving 16, 24, 32, 48 and even more permissible directions for the line segment links in the chain representation. General methods for quantization and encoding are described. The different schemes are compared with respect to compactness, precision, smoothness, simplicity of encoding, and facility for processing. The resulting coding schemes appear to have desirable characteristics for map data processing applications because of improved storage efficiency, smoothness, and reduced processing time requirements.

1. Introduction

In the chain coding scheme for the computer representation of line-drawing data, an overlaid square lattice is assumed and the lines of the drawing are represented by sequences of straight-line segments connecting nodes of the lattice lying closest to the lines. In passing from one node to the next, there are 8 allowed directions, and the concatenated line segments are all of length 1 or  $2\sqrt{2}$  (times the lattice spacing). The scheme has been found especially useful for representing free-form line drawing-data as is encountered in geographic maps. It has found wide acceptance for the purpose of digital data transmission and computer processing, mainly because of its inherent simplicity and the ease with which efficient processing algorithms can be developed for it [1,2]. We shall here show that the basic (i.e., 8-direction) chain code can be generalized to codes having a much larger number of allowed directions and that such codes, in spite of their increased complexity, may have definite advantages for certain applications [3].

In selecting a line-drawing coding scheme for a particular application, it is helpful to evaluate the scheme against the following five criteria: (1) compactness, (2) precision, (3) smoothness, (4) ease of encoding and decoding, and (5) facility for processing. The relative weight to be assigned to each of these criteria is very much dependent on the intended application. If the purpose of the

\*This research was supported by the Air Force Office of Scientific Research, Directorate of Mathematical and Information Sciences, under Grant AFOSR 76-2937.

encoding is primarily storage or transmission, compactness is likely to be of paramount importance as it directly determines the required amount of computer memory or channel bandwidth (or transmission time). Precision is important if quantitative aspects of the encoded data are of particular interest (i.e., a geographic map). Smoothness may be of significance if the encoded data is ever to be displayed, especially if the "fairness" of a curve is important or if the result is to be aesthetically pleasing.

The weight to be given to ease of encoding (and decoding) will be high if large data quantities are to be encoded. For applications involving smaller data quantities but extensive processing, simplicity of the processing task is likely to outweigh simplicity of encoding.

2. Generalized Chain Codes

In the basic (8-point) chain code, the next node  $(r, s)$  in sequence for a given present node  $(i, j)$  must be one of the 8 nodes that are 1- or  $\sqrt{2}$ -distant, i.e., such that  $\max(|r-i|, |s-j|) = 1$ . Thus in Fig. 1, for a given node A, the permissible next nodes in the basic chain code are the nodes numbered 0 through 7. All of these nodes lie on the boundary of a square of side 2 and centered at A. We shall refer to this square boundary as "ring 1".\*

Let us now consider a coding scheme in which the "next" node may be any node in ring 1 or in ring 2. (Ring 2 consists of nodes 8 through 23 in Fig. 1). These are the nodes for which  $\max(|r-i|, |s-j|) = 1$  or 2. A chain based on such a 24-point scheme may contain links of length 1,  $\sqrt{2}$ , 2,  $\sqrt{5}$ , and  $2\sqrt{2}$ . Also there will be a total of 16 allowed directions (determined by the nodes of ring 2). A curve encoded with this scheme is likely to exhibit finer angular quantization and to contain fewer segments than one encoded in the 8-point scheme. Finer angular quantization will yield improved smoothness. Fig. 2 shows a curve encoded in both the 8-point scheme (2a) and the 24-point scheme (2b).

A variety of other chain coding schemes can now be readily postulated. However, let us first look at some of the properties of the rings. Exam-

\*There is good precedent for using "ring" to denote a square entity; e.g. "boxing ring".

Approved for public release;  
distribution unlimited.

15 AFOSR-76-2937

401653

16 2304

17 A2

18 AFOSR

19 TR-78-0877

JOB

ation of Fig. 1 shows that as we advance from ring  $n$  to ring  $n+1$ , for each non-corner node of ring  $n$ , there is a corresponding non-corner node in ring  $n+1$ . For each of the 4 corner nodes of ring  $n$ , there is a corresponding corner node in ring  $n+1$  as well as 2 non-corner nodes not present in ring  $n$ . Hence the total number of nodes in ring  $n+1$  will be greater by 8 than the number of nodes in ring  $n$ . Since the number of nodes in ring 1 is 8, it follows that ring  $n$  will contain precisely  $8n$  nodes. The number of nodes for all rings, 1 through  $n$  inclusive, is  $4n(n+1)$ .

In the first octant (slope 0 through 1) the permissible slopes for the set of rings 1 through  $n$  are all those that correspond to the rational numbers between 0 and 1 inclusive whose denominators are less than or equal to  $n$ , and these, if ordered, are the terms of the Farey series of order  $n$  [4,5]. The slopes for the other octants follow from symmetry. The total number of different permissible directions for the set of rings 1 through  $n$  is given by  $8F(n) - 8$ , where  $F(n)$  is the number of terms in the Farey series of order  $n$ .

In forming a chain coding scheme, we may use any number of rings, in any combination. Thus we may form a chain code based solely on ring 2. It will have 16 permissible directions and its links will be of length 2,  $\sqrt{5}$ , and  $2\sqrt{2}$ . Its angular quantization will be either 18.4 or 26.5°. It differs from the 24-point code in that steps of length 1 or  $\sqrt{2}$  are not allowed. As a result there may be difficulty in obtaining a closed chain to correspond to a closed curve; that is, the end points of a 16-point encoded chain may be 1 or  $\sqrt{2}$  units apart without the availability of links of such lengths for closing this gap. For example, if one draws in Fig. 1 a line segment from node A to node 23 and from node 23 to node 1 (both permissible 16-point line segments), the end points, nodes A and 1, will be a distance  $\sqrt{2}$  apart. Although this lack of completeness may be objectionable to the purist, in practice it is of minor consequence since a chain can always be closed by some sacrifice in precision. Thus for the previous 2-link chain, drawing the second link from node 23 to node 9 instead of to node 1 will permit closing the chain with a link from 9 to A. The 16-link scheme has been previously proposed for use with digital plotters [6].

In returning to the 24-point code we note that (since the 8-point code is subsumed in it) it has all the features of the 8-point code of being able to follow fine detail (small radii of curvature) with segments of length 1 and  $\sqrt{2}$  but in addition has segments of length 2,  $\sqrt{5}$ , and  $2\sqrt{2}$  for "taking bigger steps" where the curvature is more gentle. These larger steps can be taken with an angular quantization roughly twice as fine as that of the 8-point scheme. Clearly, with the foregoing in mind, a 48-point scheme utilizing rings 1, 2 and 3 should be even better.

Coding matrices corresponding to 4-, 8-, 16-, 24-, 32- and 48-point codes are shown in Fig. 3. Note that the 32-point coding matrix consists of rings 1

and 3. This code thus has the ability to take relatively long, fine-angle steps but, because of ring 1, can also follow small detail. The 48-link code of Fig. 3(g) consists of the complete rings 1 and 2, and the partial ring 4. In ring 4, those nodes for which one coordinate has value 3 have been omitted. If ring 2 were also eliminated, a 32-point code would result (consisting now of ring 1 and the partial ring 4) that would have an excellent long-distance capability and yet retain the ability to follow fine detail. The rules governing the node relations for the codes in Fig. 3 are shown in Fig. 4.

### 3. Quantization

One of the appealing features of the 8-link code has been its simplicity - for quantization, for encoding, and for processing. As we go to higher-order link codes, the complexity of these tasks increases. Let us examine first the quantization problem. In Fig. 5(a), the so-called grid intersection method for the 8-point code is illustrated. One traces along the curve, and at each intersection between curve and superimposed grid, the node closest to the intersection is selected as next node. The method assures that on average approximately 41 per cent of the links in a chain will be of length  $\sqrt{2}$ . [7] An alternate quantization scheme is the so-called square-box scheme shown in Fig. 5(b), where the next node is selected on the basis of a square box "capture area" surrounding each node. The latter scheme, however, yields then only 4-point coded chains [7].

In Fig. 5(c) we show how the grid-intersection scheme has been extended to the 24-point code. In determining the next node, one first looks for the intersection between the curve and ring 2. The closest ring-2 node is identified; however, before it can be taken as the next node, it is necessary to determine whether the curve intersects ring 1 within limits set by the grid midpoints to either side of the identified ring-2 node. In Fig. 5(c), for curve A the ring-2 node is 17. Its limits in ring 1 are located at the 1/4 and 3/4 points between nodes 1 and 2 (note the dashed lines). If the curve intersects ring 1 within these limits, the ring 2 node is the valid next node. Thus in Fig. 5(c), node 17 is a valid next node for curve A, but node 9 is not a valid next node for curve B. For curve B, the next node must be taken from ring 1 (it will be node 1).

The quantization scheme for the 32-point code (based on rings 1 and 3) is shown in Fig. 5(d). Appropriate limits must be satisfied for rings 3, 2, 1 (in that order). In the figure, curve A satisfies all limits associated with node 9 and node 9 thus becomes the next node. However, node 16 cannot be selected for curve B because the associated ring-2 and ring-1 limits are not satisfied. One should note that, although the 32-point code utilizes only rings 1 and 3, for the purpose of quantization, all rings of lower order must be considered. The quantization procedure for higher-order codes is similar.



#### 4. Encoding

For the 8-point chain code, the coding convention is well known and is shown in Fig. 6 (b). In Fig. 6 (a) we show the corresponding convention for the 4-point chain code. Possible conventions for the 16- and 24-point codes are shown in Fig. 6 (c) and (d), respectively. For both of the latter codes, addition of 2 to each code value will cause a 90-degree counter-clockwise rotation (subject to appropriate limit checks to assure remaining in the same ring). Many different code assignments are, of course, possible. Proposed coding assignments for the 32-point and the two 48-point codes of Fig. 3 are shown in Figs. 7 and 8.

The number of bits required to represent a curve in the different chain-code systems varies considerably. In Fig. 9 we show a curve quantized into the 4-point, 8-point, 16-point, 24-point, and 32-point systems. If we use a distinct code word for each link, we shall require 2, 3, 4, 5, and 5 bits per link, respectively for these codes. The results are shown in Table I.

Code System	Number of Links	Bits per Link	Total No. of Bits
4	124	2	248
8	87	3	261
16	44	4	176
24	48	5	240
32	38	5	190

Table I. Bit Requirements for the Different Chains of Figure 9.

The coding assignments in Fig's. 6 through 8 all associate a unique number with each allowed line segment. A curve quantized into a chain of line segments can thus be uniquely described by a string of numbers, and such a representation implies a fixed orientation (but not position) on the coding lattice. For a curve to be "well-quantized", the change in orientation from link to link should normally be small. (A succession of many large changes in link orientation would imply that the curve was not quantized finely enough to preserve the detail of interest[8].) This suggests the presence of strong link-to-link coherence in any well-quantized chain, and this coherence can be utilized for compressing the corresponding number string. The most obvious and simple scheme is that of using first differences. For the case of the conventional 8-point chain, this leads to the so-called chain difference code [7]:

Link Diff.	Code Word	No. of Bits
0	0	2
+1	1	2
-1	2	2
+2	31	4
-2	32	4
+3	331	6
-3	332	6
INIT	3330	7
CTRL	3331	7

where the link difference "0" means continuation in

the same direction, "1" means that the succeeding link is the next one in a counterclockwise sense, etc. INIT is used, with 3 additional bits, to set the initial link direction. CTRL is a control flag, analogous to the combination 04 in the conventional chain code [2], and is also used (with additional digits) to indicate a valid link change of  $\pm 4$ .

A chain difference code for the 87-link chain

```
34312 22434 33432 21112 12232 43434 22222
21221 21111 00777 66556 56666 76767 67767
67676 66756 66554 54
```

which is illustrated in Fig. 9(c), is given by

```
+1 -1 -2 +1 0 0 +2 -1 +1 -1 0 +1 -1 -1 0
-1 0 0 +1 -1 +1 0 +1 -1 +2 -1 +1 -1 +1 -2
0 0 0 0 0 -1 +1 0 -1 +1 -1 0 0 0 -1
0 -1 0 0 -1 0 -1 0 +1 -1 +1 0 0 0 +1
-1 +1 -1 +1 -1 +1 0 -1 +1 -1 +1 -1 +1 -1 0
0 +1 -2 +1 0 0 -1 0 -1 +1 -1
```

The difference-code chain contains 81 2-bit code words (0, +1, or -1), 5 4-bit code words (+2 or -2) and, of course, an initial-direction code (INIT 3) of 10 bits to set the initial direction equal to link direction 3. The total number of bits thus is  $2(81) + 5(4) + 10 = 192$ . In comparison, the 8-point chain requires  $3(87) = 261$  bits.

Link Change	Code Word	Link Change	Code Word	No. of Bits
L+0	70	S+0	61	6
LS/SL	71	CTRL	76	6
S+1	0	S-1	1	3
L+1	2	L-1	3	3
L+2	4	L-2	5	3
L+3	72	L-3	73	6
L+4	74	L-4	75	6
L+5	62	L-5	63	6
L+6	64	L-6	65	6
S+2	66	S-2	67	6
L+7	600	L-7	601	9
L+8	602	L-8	603	9
L+9	604	L-9	605	9
L+10	606	L-10	607	9
L+11	770	L-11	771	9
S+3	772	S-3	773	9
L+12	774	S+4	775	9

#### Legend:

L+1 - next link is  $i^{\text{th}}$  long ( $>2$ ) link in positive (CCW) sense  
 S+1 - next link is  $i^{\text{th}}$  short ( $<2$ ) link in positive (CCW) sense  
 LS/SL - next link is short/long but in same direction as previous long/short link (i.e., change in link length only)

Table 2. Chain Difference Coding Scheme for 32-Point Code

The 38-link, 32-point chain of Fig. 9(f) is given by the sequence

```
19,1,10,27,11,3,2,9,25,2,11,4,3,3,10,25,2,25,9,
16,15,6,29,5,5,14,22,30,30,22,30,14,14,6,21,4,5,4
```

If written in the corresponding chain difference code, shown in Table 2, the chain will require 21 3-bit words, 15 6-bit words, 1 9-bit word, and one 12-bit initial-direction word, for a total of 174 bits. In comparison, the full chain code requires  $38(5) = 190$  bits.

### 5. Comparative Code Characteristics

We observe that for the curve of Fig. 9(a), we obtained a 26.4 per cent saving (261 to 192) when converting from the 8-point chain code to the corresponding chain difference code. For the same curve, when converting from the 32-point chain representation to the corresponding chain-difference code, the saving is only 8.4 per cent (190 to 174). The explanation, of course, is that the 32-point code is in itself much more efficient than the 8-point code and hence there is less left to be gained by going to the difference coding scheme. Essentially similar results are found for the 16-, 24-, and 48-point codes. We make the interesting observation that the number of bits required to represent a curve does not vary materially with the coding scheme used if we are prepared to use difference schemes to compress the codes. Apparently, if we are looking for advantages that will establish one code as being superior to another, we must look elsewhere than at compactness of representation.

Inspection of Fig. 9 shows that the smoothness of the representation tends to increase with increase in the order of the chain code. This is due to the increased angular resolution of the higher-order codes. When the different chains of Fig. 9 were shown to a number of observers, the majority selected the 32-point representation, (f), as the "smoothest" and most faithful rendition of the original curve given in (a). Smoothness is a largely subjective concept, difficult to quantify. The superiority of the codes of order 16 or greater over the 4- and 8-point codes is, however, quite apparent.

To obtain some measure of the relative precision for the different codes, the perimeter, enclosed area, and magnitude of area error were determined for the curve and chains of Fig. 9. Since the curve is free-form, the area had to be computed by using a secondary lattice, one fifth the size of the lattice used for the chains. The results are shown in Table 3. ("Area error" is obtained by counting both interior and exterior error as positive.)

With respect to the complexity of any processing algorithms, there is very little difference among the different chain codes. The higher-order codes merely require larger lookup tables for the algorithms; there is virtually no difference in computation time per link. However, since processing time is proportional to the number of links in a chain, the higher-order chains will, in fact, be processed much faster. Thus, for example, since the 32-point chain of Fig. 9 (f) has only 38 links whereas the corresponding 8-point chain, (c), has 87 links, the former will be processed in less than half the time required for the latter.

Code System	Perimeter	Area	Area Error
4	124.0	312.0	16.6
8	103.2	310.5	11.5
16	97.4	306.5	13.1
24	98.1	301.8	10.2
32	97.4	307.0	9.1
Curve	96.5	310.3	-

Table 3. Perimeter and Area Data Computed for the Chains of Figure 9

As described in Section 3, the quantization procedures become progressively more involved as we go to the higher-order chain codes. To a somewhat lesser extent this is also true for the "de-quantization" (i.e., display) procedures. Specialized hardware can be employed for quantization to alleviate this problem. In any case, however, the increased effort here is not likely to outweigh the significant advantages gained in smoothness, precision, and overall computation time.

### 6. Conclusion

A set of higher-order chain codes has been described which permits the use of 16, 24, 32 or even more link types for approximating a curve. The higher-order codes offer the possibility of increased precision, greater smoothness, and reduced computation time for processing line drawing data. The advantages over the well known 8-point chain code are appreciable. There is also some gain in compactness of representation; however, this is of less significance since similar gains can be obtained in other ways. The higher-order chain codes should prove especially advantageous for large-data line drawings such as geographic maps, where smoothness of representation and reduced computation time are particularly important.

### References

1. H. Freeman, "On the Encoding of Arbitrary Geometric Configurations", *IRE Trans., EC-10*, (2), June 1961, 260-268.
2. "Computer Processing of Line-Drawing Images", *Computing Surveys*, 6, (1), March 1974, 57-97.
3. "Comparative Analysis of Line-Pattern Coding Schemes", Conf. on Formal Psychophysical Approaches to Visual Perception, Nijmegen, The Netherlands, July 1976.
4. U. Montanari, "A Method for Obtaining Skeletons Using a Quasi-Euclidean Distance", *J. ACM*, 15, (4), October 1968, 600-624.
5. H. Rademacher, *Lectures on Elementary Number Theory*, Blaisdell, New York, 1964.
6. V. V. Athani, "The 16-Vector Algorithm for Computer Controlled Digital x-y Plotter", *IEEE Trans. Computers*, C-24, (8), August 1975, 831-835.
7. H. Freeman, "A Technique for the Classification and Recognition of Geometric Patterns", *Proc. 3rd Int'l. Congress on Cybernetics*, Namur, Belgium, 1961, 348-369.
8. H. Freeman and J. Glass, "On the Quantization of Line Drawing Data", *IEEE Trans. Systems Science*

and Cybernetics, SSC-5, (1), Jan. 1969, 70-79.

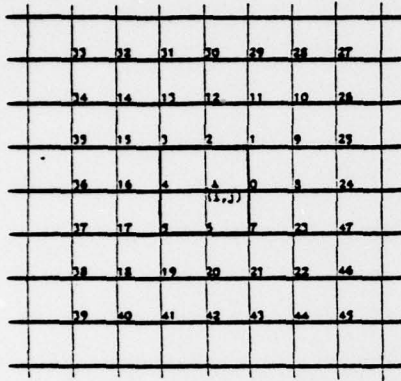


Fig. 1. The different node rings surrounding the given node A: 0 - 7 (ring 1), 8 - 23 (ring 2), 24 - 47 (ring 3), etc. Ring 1 is shown bold.

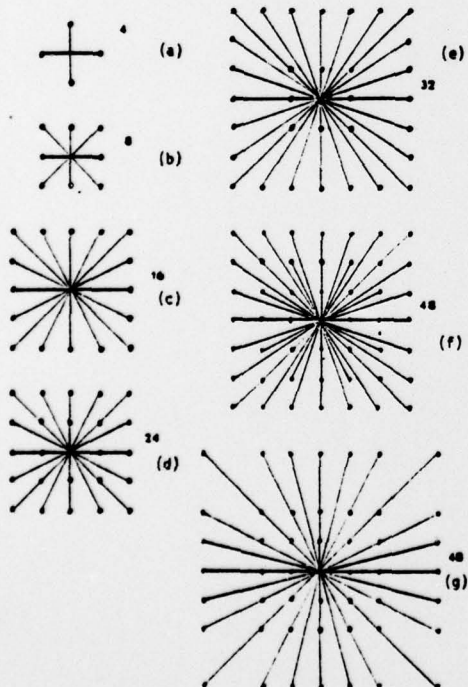


Fig. 3. Coding matrices for the 4-, 8-, 16-, 24-, 32-, and 48-point chain codes. (Two different versions of the 48-point code are shown).

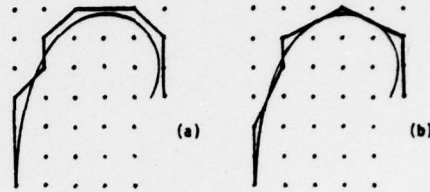
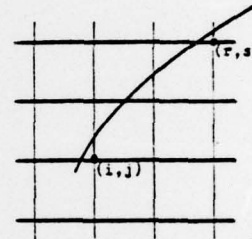


Fig. 2. Two different chain encodings of the same curve: (a) 8-point code, (b) 24-point code.



4-Point Chain:	$ r - i  +  s - j  = 1$
8-Point Chain:	$\max.  r - i ,  s - j  = 1$
16-Point Chain:	$\max.  r - i ,  s - j  = 2$
24-Point Chain:	$\max.  r - i ,  s - j  = 1 \text{ or } 2$
32-Point Chain:	$\max.  r - i ,  s - j  = 1 \text{ or } 3$
48-Point Chain: (A)	$\max.  r - i ,  s - j  = 1, 2 \text{ or } 3$
48-Point Chain:	$\max.  r - i ,  s - j  = 1, 2 \text{ or } 4$ and $ r - i  \neq 3,  s - j  \neq 3$

Fig. 4. Adjacent-node relationships for the 4-, 8-, 16-, 24-, 32-, and 48-point chain codes.

ACCESSION for	
NTIS	White Section <input checked="" type="checkbox"/>
DDC	Buff Section <input type="checkbox"/>
UNANNOUNCED	<input type="checkbox"/>
JUSTIFICATION.....	
BY.....	
DISTRIBUTION/AVAILABILITY CODES	
Dist.	AVAIL. and/or SPECIAL
A	



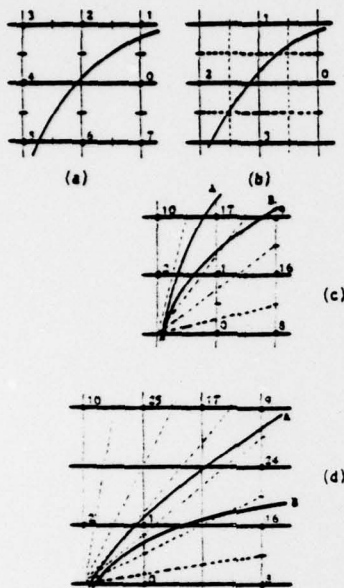


Fig. 5. Quantization schemes for the 4-, 8-, 24-, and 32-point chain codes. (a) 8-point grid-intersect quantization, (b) 4-point square-box quantization, (c) 24-point grid intersect quantization, and (d) 32-point grid-intersect quantization.

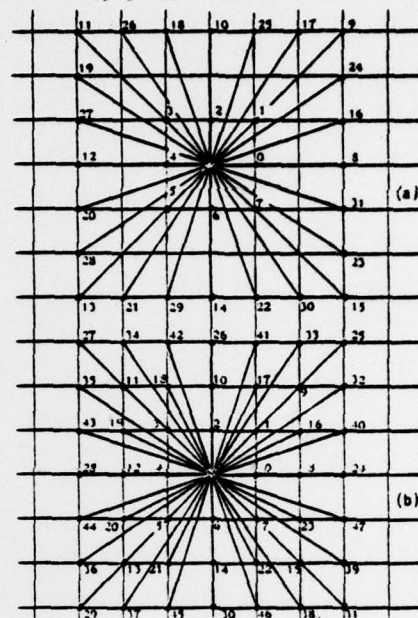


Fig. 7. Code assignments for the 32- and 48-point chain codes.

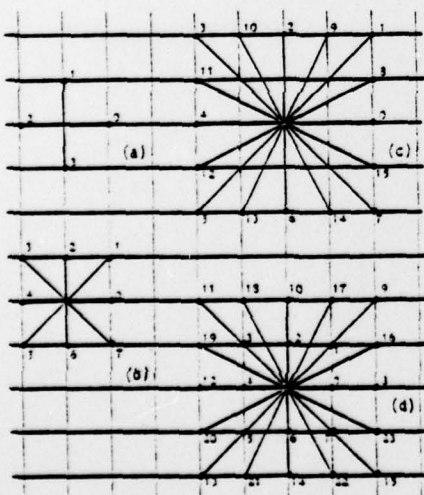


Fig. 6. Code assignments for the 4-, 8-, 16-, and 24-point chain codes.

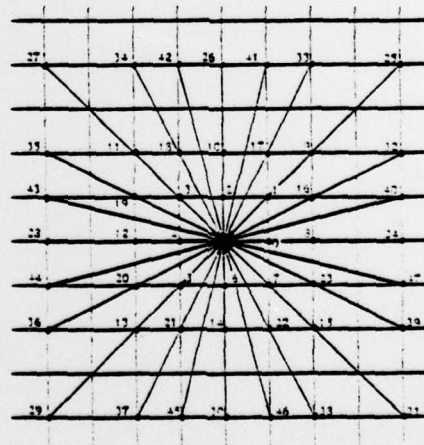


Fig. 8. 48-point chain code constituted from rings 1, 2, and a partial ring 4.

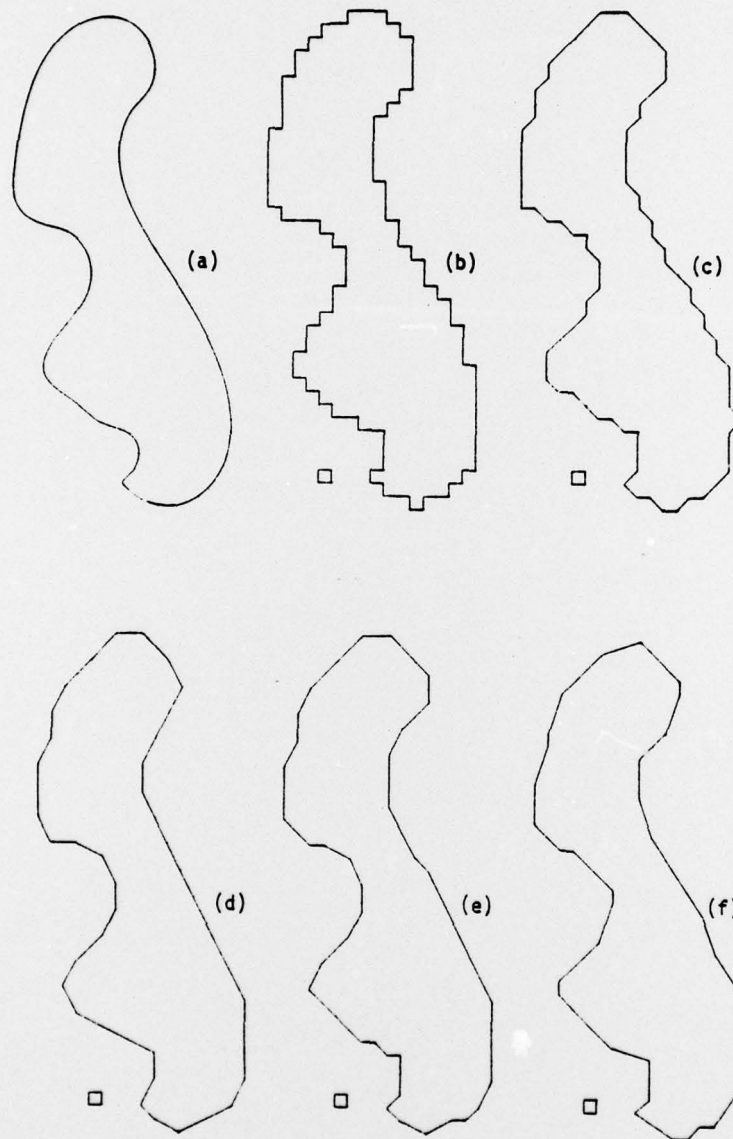


Fig. 9. Comparison of original contour (a) with different chain types: (b) 4-point, (c) 8-point, (d) 16-point, (e) 24-point, and (f) 32-point chain.



UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER <b>AFOSR-TR- 78 - 0877</b>	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) APPLICATION OF THE GENERALIZED CHAIN CODING SCHEME TO MAP DATA PROCESSING		5. TYPE OF REPORT & PERIOD COVERED  Interim
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Herbert Freeman		8. CONTRACT OR GRANT NUMBER(s) AFOSR 76-2937
9. PERFORMING ORGANIZATION NAME AND ADDRESS Rensselaer Polytechnic Institute Department of Electrical & Systems Engineering Troy, New York 12181		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS  61102F 2304/A2
11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Office of Scientific Research/NM Bolling AFB, DC 20332		12. REPORT DATE 1978
		13. NUMBER OF PAGES 7
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report)  Approved for public release; distribution unlimited,		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) line drawing processing cartography image processing pattern recognition map data encoding		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number)  The concept of chain coding based on the well known 8-direction coding matrix is generalized to coding schemes involving 16, 24, 32, 48 and even more permissible directions for the line segment links in the chain representation. General methods for quantization and encoding are described. The different schemes are compared with respect to compactness, precision, smoothness, simplicity of encoding, and facility for processing. The resulting coding schemes appear to have desirable characteristics for map data processing applications because of improved storage efficiency, smoothness, and reduced		

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

20. Abstract

processing time requirements.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)